PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

NOVEMBER 14TH, 1853.

HUMPHREY LLOYD, D. D., VICE-PRESIDENT, in the Chair.

THE Secretary announced the bequest of books and manuscripts made to the Academy by the late William Elliott Hudson, Esq., M. R. I. A.; and also the donation of his Bust by MOORE, presented by his Executors.

RESOLVED,—That the thanks of the Academy be presented to the Executors of the late William E. Hudson, Esq., for their donation of his Bust, and that the Academy entertain the highest sense of the value of the bequest left by Mr. Hudson to the Academy.

RESOLVED,—That the bust of the late W. E. Hudson, Esq., be placed in the Library, as a mark of respect to his memory, and of gratitude for the bequest with which he has enriched the Library of the Academy.

The Rev. Charles Graves, D. D., read a paper on the solution of linear differential equations.

The object of the present paper is to contribute to our knowledge of the soluble forms of linear differential equations. The left-hand member of the equation

$$D^n y + AD^{n-1}y + By \ldots = X$$

in which A, B, &c., are any functions of x, being regarded as the result of the operation of

$$D^n + AD^{n-1} + .. B$$

upon y, all attempts to solve it by the method of the separation of symbols must be directed to the transformation of this operator into a form in which it appears as the product of a series of operations, each of which admits of being inverted; and, conversely, if the result of a series of such operations be to produce a complex operation of the preceding form, we may apply it to the construction of soluble forms of differential equations.

Thus the operation

$$(D + \phi)\,(D + \psi),$$

in which ϕ and ψ denote any functions of x, will be found on development to be equal to

$$D^2 + (\phi + \psi)\,D + (\phi\psi + \psi'\,);$$

consequently, we may identify the general linear differential equation of the second order,

$$D^2y + ADy + By = X \qquad (1)$$

with $$D^2y + (\phi + \psi)\,Dy + (\phi\psi + \psi')y = X. \qquad (2)$$

And if we could succeed in solving the equations

$$\phi + \psi = A,\ \phi\psi + \psi' = B, \qquad (3)$$

and so obtaining finite values of ϕ and ψ in terms of A and B, we should be able to effect the solution of the general linear differential equation of the second order, at least in a symbolic form; for as

$$(D + \phi)^{-1} = e^{-\int\phi dx}\int e^{\int\phi dx}$$

and

$$(D + \psi)^{-1} = e^{-\int\psi dx}\int e^{\int\psi dx},$$

we should have

$$y = e^{-\int\psi dx}\int e^{\int(\psi-\phi)dx}\int e^{\int\phi dx}X.$$

Unfortunately it happens that, in trying to determine ϕ and ψ from the equations (3), we either obtain a differential equation of the first order and *second* degree, or are led back again to the solution of the equation (1).

We may, however, make most advantageous use of the equation (2) by assigning arbitrary forms to the functions ϕ and ψ contained in it, and so construct soluble forms *ad libitum*.

When the equation (1) wants the term involving Dy, we have

$$\phi + \psi = 0 \text{ and } \phi\psi + \psi' = \text{B}.$$

Hence
$$-\phi^2 - \phi' = B. \qquad (4)$$

Now, as the second term of the equation (1) can always be banished by a change of the dependent variable, we have arrived at the remarkable result that the solution of the general linear differential equation of the second order depends upon that of the equation (4), whose form is particular and unchanging: and this result is practically important; for if we tabulate the values of $\phi^2 + \phi'$ for all values of ϕ, we should have the solutions themselves of linear differential equations of the second order tabulated at the same time.

By interchanging the symbols x and D in the preceding formulæ, according to the method pointed out by Dr. Hargreave, we are led to a series of general and interesting results.

Dr. Todd made some remarks on the fresco painting in the Abbey of Knockmoy, in the county Galway, of which a fac-simile copy, the exact size of the original, was exhibited in the Antiquarian Court of the Dublin Exhibition.

The public are indebted for the preservation and exhibition of this ancient monument of Irish art to the zeal of Dr. John Lentaigne, at whose instance, and by whose personal exertions, the fac-simile was obtained for the Committee of the Exhibition. The following account of the manner in which the inscriptions were deciphered is given in a letter dated 13th June, 1853, addressed to Dr. Todd by Mr. Eugene Curry:

"John Lentaigne, Esq., on the part of the Committee of the Great Industrial Exhibition, having done me the honour to request me to accompany him to the ruins of the once